DUSO Mathematics League 2015 - 2016

Contest #5.   Calculators are not permitted on this contest.

Part I.   **ALGEBRA I**   *Time Limit: 10 minutes*
The word "compute" calls for an exact answer in simplest form.

5 - 1. In a case are small packages and large packages of bubble gum. Small packages have 3 pieces of gum, and large packages have 5 pieces of gum. There are 20 packages in the case, and there is a total of 72 pieces of gum. Compute the number of large packages of gum in the case.

5 - 2. When expanded, $\left(\sqrt{7} - \sqrt{2}\right)^4$ is equal to $A - B\sqrt{14}$. Compute $(A - B)^2$.

Part II.   **GEOMETRY**   *Time Limit: 10 minutes*
The word "compute" calls for an exact answer in simplest form.

5 - 3. The points $(3, 1995)$ and $(-2, 2015)$ lie on a line. The point $(10, M)$ also lies on this line. Compute M.

5 - 4. Quadrilateral $QUAD$ has all four vertices on a circle O. The lengths of the sides of $QUAD$ are $QU = 5$, $UA = \sqrt{43}$, $AD = 3\sqrt{5}$, and $DQ = 3\sqrt{7}$. Compute the area of circle O.

Part III.   **ALGEBRA II / ADVANCED TOPICS**   *Time Limit: 10 minutes*
The word "compute" calls for an exact answer in simplest form.

5 - 5. The solution to $2^{x-3} = 45$ can be expressed as $\dfrac{\ln A}{\ln B}$ where A and B are natural numbers and $\dfrac{\ln A}{\ln B}$ is in simplest form. Compute the ordered pair (A, B).

5 - 6. Regular hexagon $ABCDEF$ has side length 1. Square $ABGH$ is constructed exterior to $ABCDEF$. The length CG can be expressed in simplest form as $\dfrac{\sqrt{P} + \sqrt{Q}}{R}$ for some natural numbers P, Q, and R with $P < Q$. Compute CG in this form.

R-1. Compute the least odd positive integer that is the product of four distinct prime numbers.

R-2. *Let N be the number you will receive.* The quadratic equation $x^2 - 2x = N$ has two roots. Compute the greater of the two roots.

R-3. *Let N be the number you will receive.* In the sequence $41, N, ...,$ the difference between any two consecutive terms is constant. Compute the sixth term in the sequence.

R-4. *Let N be the number you will receive.* A right circular cylinder has a height of N cm. The surface area of the cylinder (including its top and its bottom) is 120π square cm. Compute the radius of the base of the cylinder in cm.

R-5. *Let N be the number you will receive.* Consider the set $S = \{k, 9, 10, 16, 19, N\}$, where the elements of S are integers. The mean of the numbers in S is 2 more than the median number in S. Compute k.